Griffiths

5.10

$\alpha \approx \frac{1}{137}$, $mc^2 \approx 0.5$ MeV for electron.

$j = \frac{3}{2}$, $\frac{1}{4n^2}\left(\frac{2n}{(j+\frac{1}{2})} - \frac{3}{2}\right) = \frac{1}{16}\left(\frac{4}{2} - \frac{3}{2}\right)$

$= \frac{1}{16}\left(\frac{1}{2}\right) = \frac{1}{32}$.

$j = \frac{1}{2}$, $\frac{1}{4n^2}\left(\frac{2n}{j+\frac{1}{2}} - \frac{3}{2}\right) = \frac{1}{16}\left(\frac{4}{2} - \frac{3}{2}\right)$

$= \frac{1}{16}\left(\frac{5}{2}\right) = \frac{5}{32}$.

For $j = \frac{3}{2}$, $\Delta E \approx -\frac{1}{(137)^4}\frac{1}{32} \times 5 \times 10^5$ eV

$\approx \boxed{-0.000044 \text{ eV}}$

For $j = \frac{1}{2}$, $\Delta E \approx \boxed{-0.00022 \text{ eV}}$.

Compared to $n=1$, $n=2$ Bohr energies (13.6eV, 3.4eV), these are extremely small.